## Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

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| CANDIDATE <br> NUMBER |  |  |  |  |
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## ADDITIONAL MATHEMATICS

4037/12
Paper 1
October/November 2022

You must answer on the question paper.
No additional materials are needed.

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.


## INFORMATION

- The total mark for this paper is 80 .
- The number of marks for each question or part question is shown in brackets [ ].


## Mathematical Formulae

## 1. ALGEBRA

Quadratic Equation
For the equation $a x^{2}+b x+c=0$,

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a}
$$

Binomial Theorem

$$
(a+b)^{n}=a^{n}+\binom{n}{1} a^{n-1} b+\binom{n}{2} a^{n-2} b^{2}+\ldots+\binom{n}{r} a^{n-r} b^{r}+\ldots+b^{n}
$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$

Arithmetic series

$$
\begin{aligned}
& u_{n}=a+(n-1) d \\
& S_{n}=\frac{1}{2} n(a+l)=\frac{1}{2} n\{2 a+(n-1) d\}
\end{aligned}
$$

Geometric series

$$
\begin{aligned}
& u_{n}=a r^{n-1} \\
& S_{n}=\frac{a\left(1-r^{n}\right)}{1-r}(r \neq 1) \\
& S_{\infty}=\frac{a}{1-r}(|r|<1)
\end{aligned}
$$

## 2. TRIGONOMETRY

Identities

$$
\begin{gathered}
\sin ^{2} A+\cos ^{2} A=1 \\
\sec ^{2} A=1+\tan ^{2} A \\
\operatorname{cosec}^{2} A=1+\cot ^{2} A
\end{gathered}
$$

Formulae for $\triangle A B C$

$$
\begin{gathered}
\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C} \\
a^{2}=b^{2}+c^{2}-2 b c \cos A \\
\Delta=\frac{1}{2} b c \sin A
\end{gathered}
$$

1


The diagram shows the graph of $y=a \sin b x+c$, where $a, b$ and $c$ are integers. Find the values of $a, b$ and $c$.

2 (a) On the axes, draw the graph of $y=\left|3 x^{2}+13 x-10\right|$, stating the coordinates of the points where the graph meets the axes.

(b) Find the set of values of the constant $k$ such that the equation $k=\left|3 x^{2}+13 x-10\right|$ has exactly 2 distinct roots.

3 Write $\frac{\sqrt{\left(9 p^{2} q\right)} \times r^{-3}}{(2 p)^{3} q^{-1} \sqrt[5]{r}}$ in the form $k p^{a} q^{b} r^{c}$, where $k, a, b$ and $c$ are constants.

4 Solve the equation $3 \sin \left(2 x+\frac{\pi}{4}\right)=\sqrt{3} \cos \left(2 x+\frac{\pi}{4}\right)$, for $0 \leqslant x \leqslant \pi$.

5 (a) Find the vector with magnitude 200 in the direction of $\binom{7}{-24}$.
(b)


The diagram shows triangle $A O B$ such that $\overrightarrow{O A}=\mathbf{a}$, and $\overrightarrow{O B}=\mathbf{b}$. The point $C$ lies on the line $A B$ such that $A C: A B=1: 3$. Find the vector $\overrightarrow{O C}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, giving your answer in its simplest form.
(c) Given the vector equation $p\binom{2}{1}+q\binom{2}{4}=5\binom{-p+1}{p+q}$, find the values of $p$ and $q$.

6 A group of 15 people includes 3 brothers. A team of 6 people is to be chosen from this group. The three brothers must not be separated. Find the number of possible teams that can be chosen.


The diagram shows a circle, centre $O$, radius 10 cm . The points $A$ and $B$ lie on the circumference of the circle. The tangent at $A$ and the tangent at $B$ meet at the point $C$. The angle $A O B$ is $\theta$ radians. The length of the minor arc $A B$ is 28 cm .
(a) Find the value of $\theta$.
(b) Find the perimeter of the shaded region.
(c) Find the area of the shaded region.

8 A function $\mathrm{f}(x)$ is such that $\mathrm{f}(x)=\ln (2 x+3)+\ln 4$, for $x>a$, where $a$ is a constant.
(a) Write down the least possible value of $a$.
(b) Using your value of $a$, write down the range of f .
(c) Using your value of $a$, find $\mathrm{f}^{-1}(x)$, stating its range.
(d) On the axes below, sketch the graphs of $y=\mathrm{f}(x)$ and $y=\mathrm{f}^{-1}(x)$, stating the exact intercepts of each graph with the coordinate axes. Label each of your graphs.


9 (a) Show that $\frac{1}{2 x+1}-\frac{1}{(2 x+1)^{2}}+\frac{4}{4 x-1}=\frac{24 x^{2}+14 x+4}{(2 x+1)^{2}(4 x-1)}$.
(b) Hence find $\int_{\frac{1}{2}}^{1} \frac{24 x^{2}+14 x+4}{(2 x+1)^{2}(4 x-1)} \mathrm{d} x$, giving your answer in the form $\frac{1}{2} \ln p+q$, where $p$ and $q$ are rational numbers.

10 The first three terms of an arithmetic progression are $\lg x, \lg x^{5}, \lg x^{9}$, where $x>0$.
(a) Show that the sum to $n$ terms of this arithmetic progression can be written as $n(p n-1) \lg x$, where $p$ is an integer.
(b) Hence find the value of $n$ for which the sum to $n$ terms is equal to $4950 \lg x$.
(c) Given that this sum to $n$ terms is also equal to -14850 , find the exact value of $x$.

11 A particle $P$ moves in a straight line such that, $t$ seconds after passing through a fixed point $O$, its displacement, $s$ metres, is given by $s=\frac{(2 t+1)^{\frac{3}{2}}}{t+1}-1$.
(a) Show that the velocity of $P$ at time $t$ can be written in the form $\frac{(2 t+1)^{\frac{1}{2}}}{(t+1)^{2}}(a+b t)$, where $a$ and $b$
are integers to be found.
(b) Show that $P$ is never at instantaneous rest after passing through $O$.

12 The first three terms, in descending powers of $x$, of the expansion of $\left(a x+\frac{2}{5}\right)^{5}\left(1-\frac{b}{x}\right)^{2}$, can be written as $32 x^{5}-160 x^{4}+c x^{3}$, where $a, b$ and $c$ are constants. Find the exact values of $a, b$ and $c$.

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